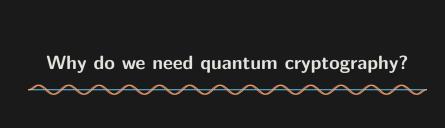
The quest for secure quantum communication

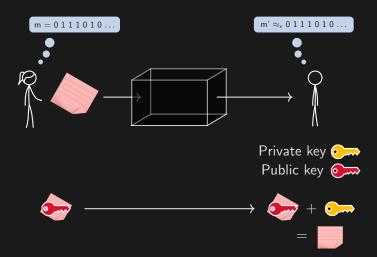


Ramona Wolf, University of Siegen



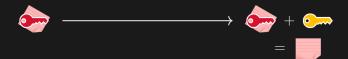


A glimpse of classical cryptography RSA encryption



Is this secure?

What makes RSA secure?



p, q randomly chosen, large prime numbers

Knowledge of $p, q \Rightarrow Can$ calculate the private key



Why is this secure?

⇒ Factoring large numbers is hard

Not for a quantum computer!

General problems in classical cryptography

We do not know if there exists an algorithm that breaks the encryption (classical or quantum)

encryption can be broken anytime

"Store now, decrypt later"
attacks make quantum
computers already
a security
threat today

Computational power (classical or quantum) increases over time

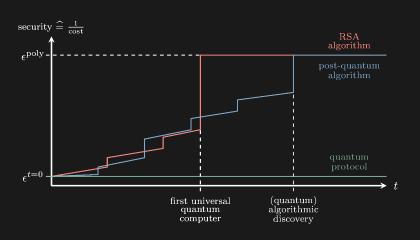


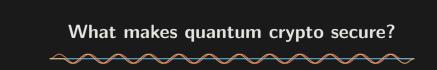


| Digits | Number of operations | Time |
|--------|------------------------|----------------------------|
| 50 | 1.4 × 10 ³⁰ | 3.9 hours |
| 75 | 9.0×10^{12} | 104 days |
| 100 | 2.3×10^{15} | 74 years |
| 200 | 1.2×10^{23} | 3.8×10^9 years |
| 300 | 1.5×10^{29} | 4.9 × 1015 years |
| 500 | 1.3×10^{39} | 4.2×10^{25} years |

Security over time

Classical vs. quantum protocols





Classical cryptography

Assumptions

Security proof

Security guarantee

Factoring large numbers is hard

If you could break RSA, you could factor large numbers RSA encryption cannot be broken

Classical cryptography

Assumptions

Security proof

Security guarantee

is hard (restriction on the adversary) Breaking encryption violates the assumption

The encryption scheme cannot be broken

Quantum cryptography

Assumptions

Security proof

Security guarantee

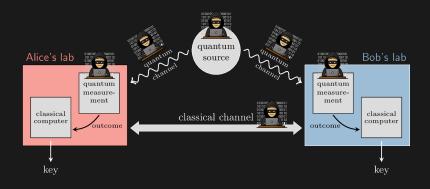
Quantum physics (+ ...)

Breaking The encryption scheme cannot violates quantum mechanics

How does QKD work?

QKD protocol

The setup



QKD protocol

Steps

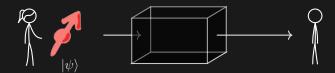
Quantum phase:

- 1. For *n* rounds: Quantum states are distributed to Alice and Bob
- 2. Alice and Bob measure the states
- 3. Alice and Bob have raw keys A_1, \ldots, A_n and B_1, \ldots, B_n

Classical post-processing:

- 4. Alice and Bob estimate the knowledge of a possible eavesdropper (via Bell violation, errors in the bit strings,...) Important quantity: $H_{\min}^{\varepsilon}(A_1,\ldots,A_n|E)$
- 5. Error correction: Turn raw keys into identical bit strings
- 6. Privacy amplification: Remove the adversary's knowledge

QKD protocol Why does this work?



QKD protocol

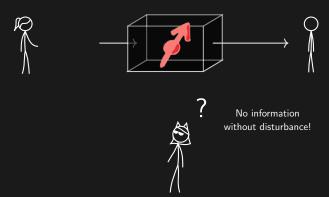
Why does this work?



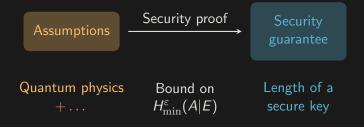


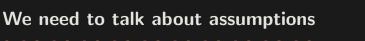
QKD protocol

Why does this work?



Quantum cryptography





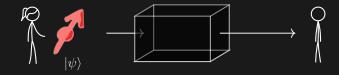
Assumptions on QKD

What do we trust?

Security proof Assumptions Quantum physics Bound on Length of a $H_{\min}^{\varepsilon}(A|E)$ secure key Mathematical model of the devices Alice and Bob's labs are closed

What is information?

What is information?

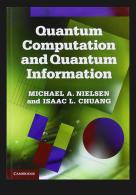


"Information is Physical"

- R. Landauer

Quantum information theory

Nielsen-Chuang

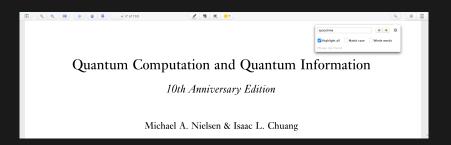


- P1: Isolated system $S o \mathsf{Hilbert}$ space \mathcal{H}
- P2: State of $\mathcal{S} o$ norm-1 vector $\psi \in \mathcal{H}$
- P3: Isolated evolution \rightarrow unitary $\psi_2 = U\psi_1$

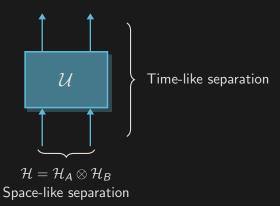
P4: Meas.
$$\rightarrow P(m) = \langle \psi | M_m^{\dagger} M_m | \psi \rangle$$

$$\psi_{m} = \frac{M_{m}|\psi\rangle}{\sqrt{P(m)}}$$

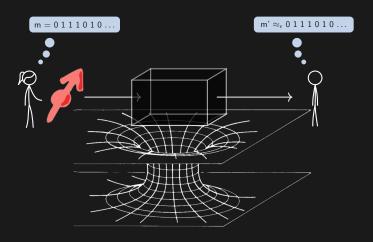
Nielsen-Chuang



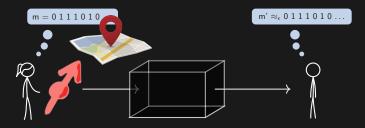
(abstracted to a simple partial order notion)



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(abstracted to a simple partial order notion)



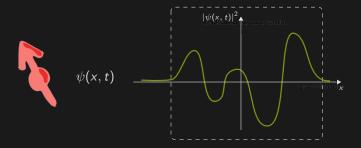
"Information is physical"

Physical systems occupy a region of spacetime

 \rightarrow Information occupies a region of spacetime

Where is information localised?

Location of particles



For any t, it's the smallest region where the particle is found with certainty

Location of information?

Locating a bit encoded in one particle

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|0\rangle \, \pm \, |1\rangle\right)$$

$$\pm \qquad - \text{enc} \longrightarrow$$

Location: same as particle 1

Locating a bit encoded in two particles

$$|\psi
angle=rac{1}{\sqrt{2}}\left(|01
angle \pm |10
angle
ight)$$



Location: same as particles 1 and 2

Locating a bit encoded in two particles

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|01\rangle \, \stackrel{\textstyle \perp}{=} \, |10\rangle \right)$$



What if we can only access relative degrees of freedom?

Quantum reference frames (QRF)

If QM is universally valid, systems that are used as a reference are ultimately quantum mechanical



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If QM is universally valid, systems that are used as a reference are ultimately quantum mechanical

Taking the perspective of 1:

$$|x_1\rangle_1|x_2\rangle_2 \mapsto |x_2-x_1\rangle_{2|1}$$



Locating a bit encoded in two particles

Taking the perspective of 1

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|10\rangle \pm |01\rangle)$$



Location: same as particles 1 and 2

Locating a bit encoded in two particles

Taking the perspective of 1

$$|\psi\rangle_{2|1} = \frac{1}{\sqrt{2}} \left(|1\rangle \boxed{\pm} |-1\rangle \right)$$



Location: same as particles 1 and 2

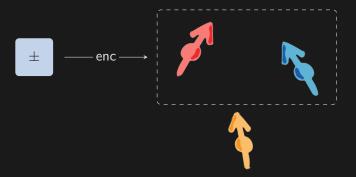
Taking the perspective of 1

$$|\psi\rangle_{2|1} = \frac{1}{\sqrt{2}} (|1\rangle \pm |-1\rangle)$$

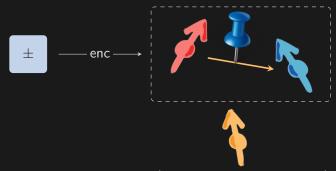


Location: same as particles 1 and 2 (specifically in the relative dof)

$$|\psi\rangle_{123} = \frac{1}{\sqrt{2}} \left(|01\rangle \pm |10\rangle \right) |0\rangle$$



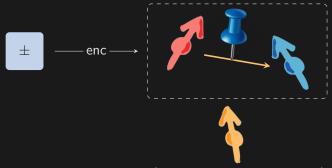
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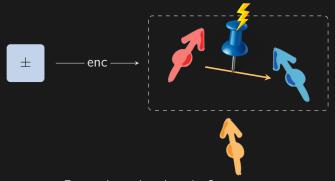
$$|\psi
angle_{23|1}=rac{1}{\sqrt{2}}\left(|10
angle\;\pm\;|-1-1
angle
ight)$$



Location: same as particles 1 and 2 (specifically in the relative dof)

Taking the perspective of 1

$$|\psi
angle_{23|1}=rac{1}{\sqrt{2}}\left(|10
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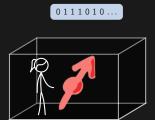
Frame-dependent location?



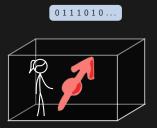
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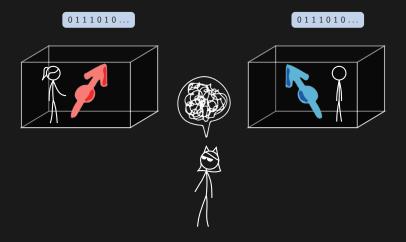
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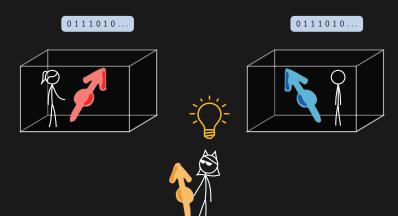












Conclusion

Conclusion

Assuming the universality of quantum mechanics, information is not objectively localised.

→ How does this affect cryptography?

Look for concrete attacks that would exploit features of quantum reference frames and adapt security.

 \rightarrow More general: Assumptions in QKD

How can we bring the theoretical models and practical implementations closer together? What are reasonable assumptions?

